Randomness In The Higher Setting

(Joint Work With André Nies and Liang Yu)

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Recursion Theory and Π^1_1-ness

- (Spector) $x \in 2^\omega$ is "r.e." over $L_{\omega_1^{\mathrm{CK}}} \Leftrightarrow x$ is Π^1_1.
- (Spector and Gandy) $A \subseteq 2^\omega$ is $\Pi^1_1 \Leftrightarrow$ there is a Σ_0-formula φ such that $x \in A \Leftrightarrow L_{\omega_1^x}!x] \models \exists z \varphi(z, x)$.
  Or equivalently, A is Π^1_1 iff there is an e such that

  $$x \in A \Leftrightarrow \{e\}^X \text{ is a well-ordering.}$$

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Notions of Higher Randomness

Randomness over $L_{\omega_1^{\mathrm{CK}}}$:

- x is Π^1_1-random (Δ^1_1-random) if x is not in any Π^1_1 (Δ^1_1)-null subset of 2^ω.
- x is Π^1_1 Martin-Löf random (Δ^1_1 Martin-Löf random) if $x \notin \bigcap_n V_n$ for any Π^1_1 (Δ^1_1)-collection of open sets $\{V_n\}$ such that $\mu(V_n) \leq 2^{-n}$.

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Higher Randomness Notions

- (Hjorth and Nies) Every hyperdegree above the degree of Kleene's $\mathcal{O}$ is the degree of a Π^1_1 ML-random real. Hence Π^1_1 ML-random reals are cofinal in the hyperdegrees.
- (Chong, Nies and Yu)
  - No $x \geq_h \mathcal{O}$ is Π^1_1-random. Hence if x is Π^1_1-random, then $\omega_1^x = \omega_1^{\mathrm{CK}}$.
  - Δ^1_1-randomness = Δ^1_1 ML-randomness
  - If $\omega_1^x = \omega_1^{\mathrm{CK}}$, then for x, Π^1_1 -random $=$ Π^1_1 ML-random $=$ Δ^1_1 -random
  - In general, Π^1_1-random $\Rightarrow$ Π^1_1 ML-random $\Rightarrow$ Δ^1_1-random, and the arrows do not reverse.

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Hyperdegrees of Random Reals

Theorem

Let $\omega_1^x = \omega_1^{\mathrm{CK}}$.

1. *If x is Δ_1^1-random, then there is a $y >_h x$ whose hyperdegree contains no Δ_1^1-random real.*
2. *If x is random (in any sense defined), then every non-hyperarithmetic hyperdegree below $\deg(x)$ contains a random real.*

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Lowness for Randomness

Definition. A real y is *low* for Π^1_1 ML-randomness if for any Π^1_1ML-random x, $x \notin \bigcap_n V_n$ for any $\Pi^1_1(y)$-collection of open sets $\{V_n\}$ such that $\mu(V_n) \leq 2^{-n}$.

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Definition. A real y is *low* for Π^1_1 (Δ^1_1)-randomness if no Π^1_1 (Δ^1_1)-random real x belongs to a $\Pi^1_1(y)$ ($\Delta^1_1(y)$)-null set.

Theorem

There is an uncountable Σ^1_1-set of reals that are low for Δ^1_1-randomness.

Corollary. "Low for Π^1_1 ML-randomness" is not equivalent to "low for Δ^1_1-randomness".

- Problem: Characterize the set of reals that are low for Π^1_1-randomness. In particular, is there a non-hyperarithmetic real that is low for Π^1_1-randomness?

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Separating Low For Randomness Between Π^1_1 And Δ^1_1

- (Martin: Proof of Friedman's conjecture) Let A be an uncountable Δ^1_1-set of reals. Then

$$\{\deg(x) : x \in A\} \supseteq \{\mathbf{z} : \mathbf{z} \geq \deg(\mathcal{O})\}.$$

Theorem

Let A_0 and A_1 be Σ^1_1 and uncountable. Then

$$\{deg(x_0 \oplus x_1) : x_0 \in A_0 \wedge x_1 \in A_1\} \supseteq \{\mathbf{z} : \mathbf{z} \geq \deg(\mathcal{O})\}.$$

Proof. Given $z \geq_h \mathcal{O}$, construct $x_0 \in A_0, x_1 \in A_1$ so that x_0 codes z and a $z_0 \equiv_h \mathcal{O}$ as well as a Σ^1_1 witness for x_1, and x_1 codes a Σ^1_1 witness for x_0.

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Separating Low For Randomness Between Δ^1_1 And Π^1_1

Corollary. There is an x which is low for Δ^1_1-randomness but not low for Π^1_1-randomness.

Proof. Harrington, Nies and Slaman showed that a real x is low for Π^1_1-randomness if and only if it is low for Δ^1_1-randomness and there is no Π^1_1-random y such that $x \oplus y \geq_h \mathcal{O}$.

There are uncountable Σ^1_1-sets A_0 and A_1 such that every real in A_0 is Π^1_1-random and every real in A_1 is low for Δ^1_1-random. Then $\mathcal{O}$ is the hyperdegree of the joint of some $x_0 \in A_0$ and x_1 in A_1. Then x_1 its low for Δ^1_1-randomness but not low for Π^1_1-randomness.

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A Notion Stronger Than Π^1_1 ML-Randomness

Definition. x is *strongly* Π^1_1 ML-random if for any $\{V_n\}_{n<\omega}$ such that $\lim_n \mu(V_n) = 0$, $x \notin \bigcap_n V_n$.
In the literature (for first-order randomness), x is known as a weakly 2-random real.

Theorem

(Also by Bievenu, Greenberg and Monin) *There is an x which is Π^1_1 ML-random but not strongly Π^1_1 ML-random.*

Proof. There is a Σ^1_1-closed tree $T \subset 2^{<\omega}$ such that $[T]$ is uncountable consisting only of Π^1_1 ML-random reals. The leftmost path of T (which is $\geq_h \mathcal{O}$) is not strongly Π^1_1 ML-random.

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If $\mathbf{z} \geq \deg(\mathcal{O})$*, then* $\mathbf{z}$ *contains a real that is* Π^1_1 *ML-random but not strongly* Π^1_1 *ML-random.*

Proof. Construct an $\mathcal{O}$-recursive perfect tree T (over $L_{\omega_1^{\mathrm{CK}}}$) such that each path on the tree ω_1^{CK}-recursively computes $\mathcal{O}$ and is Π^1_1 ML-random but not strongly Π^1_1 ML-random.

Conjecture. No real $\geq_h \mathcal{O}$ is strongly Π^1_1 ML-random.
Question. Is Π^1_1-randomness different from strong Π^1_1 ML-randomness?

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Theorem

If $\mathbf{z} \geq \deg(\mathcal{O})$*, then* $\mathbf{z}$ *contains a real that is* Π_1^1 *ML-random but not strongly* Π_1^1 *ML-random.*

Proof. Construct an $\mathcal{O}$-recursive perfect tree T (over $L_{\omega_1^{\mathrm{CK}}}$) such that each path on the tree ω_1^{CK}-recursively computes $\mathcal{O}$ and is Π_1^1 ML-random but not strongly Π_1^1 ML-random.

Conjecture. No real $\geq_h \mathcal{O}$ is strongly Π_1^1 ML-random.
Question. Is Π_1^1-randomness different from strong Π_1^1 ML-randomness?