Computable invariant measures and algorithmically random structures

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Given a countable structure, when is a presentation of it algorithmically random? Computable invariant measures concentrated on the isomorphism class of the structure provide one possible approach to this question, as suggested by Fouché and Nies (Logic Blog 2012). But when there are many such invariant measures, there may not be a single natural choice — leading to the question of when there is a unique such invariant measure.

In joint work with Ackerman, Kwiatkowska, and Patel, we show that the isomorphism class of a countable structure in a countable language admits a unique S_∞-invariant probability measure if and only if, for each n, it realizes a unique n-type up to permutation. Such a structure is called *highly homogeneous*; this notion arose in Cameron's 1976 classification of the reducts of the rational linear order $(\mathbb{Q}, <)$. In particular, there are five such structures, up to interdefinability, each of whose unique invariant measures has a computable presentation. Furthermore, we show that any countable structure admitting more than one invariant measure must admit continuum-many ergodic invariant measures.

Invariant measures on relational structures can be naturally described in terms of sampling procedures from certain measurable objects, as essentially shown by Aldous and Hoover. This representation is used in the proof of the above result about unique invariant measures, and also plays an important role in Bayesian nonparametric statistics. In joint work with Avigad, Roy, and Rute, we also address the question of when the sampling procedure corresponding to a computable invariant measure can be given in terms of a computable such object.