An introduction to Special Session on Professor Tanaka's Works

(His works and me)

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壽
恭祝
田中一之先生
六十壽辰
中國學生
劉晨光 彭寧寧
彭偉光 李文娟

Prof. Tanaka's Personal History

1955 Born in Tokyo

1978 Bachelor in Tokyo Institute of Technology

1980 Master in Tokyo Institute of Technology

1986 Ph.D in University of California, Berkeley (supervisor: Leo A. Harrington)

1986 Research Associate in Tokyo Institute of Technology

1991 Associate Professor in Tohoku University

1997 Professor in Tohoku University

Works

He has published more than 40 papers and 11 books (including 4 translations) and edited a serieses.

His articles can be mainly classified into 4 categories according to subjects:

1. Models of Arithmetics (12%): T. Wong
2. Complexity of diverse computational objects (And-Or trees etc.) (17%): T. Suzuki
3. Reverse Mathematics on Infinite Games (25%) : K.Yoshii
4. Others: Reverse Mathematics etc

Memories of his first works

In 1997, Tanaka published his self-embedding theorem in "The self-embedding theorem of WKL_0 and a non-standard method" in **Annals of Pure and Applied Logic 84**.

Theorem 1 (the self-embedding theorem)**.** *Let* (M, S) *be a countable non-standard model of* WKL_0*. Then there exists a proper initial segment* $I \subseteq M$ *such that* (M, S) *is isomorphic to* $(I, S\lceil I)$*, where* $S\lceil I = \{X \cap I : X \in S\}$*.*

Using the theorem and exchanging the role of (M, S) and the initial part, we can say that any countable non-standard model of WKL_0 has a "non-standard universe". We can declare that this theorem and its application is one of his most innovative results.

By this method, he gave non-standard proofs of theorems derived from WKL_0 one after another: the maximum principle, Riemann integrability, Peano existence's theorem etc.

Along a series of these results, Tanaka and I showed

Theorem 2 (Tanaka and Y. 00)**.** *The following are equivalent to each other over* RCA_0

(1) WKL_0

(2) *Any compact group G has a Haar measure, more prescicely, a σ-additive left-invariant positive linear operator μ on the space $C(G)$ such that $\mu(1) = 1$.*

Memories: At that time, he tried to formalize and modify many kinds of its proof. When he found a non-standard proof by Hauser and others, he must have been certain of our success. A simple idea told us that his prospect is true.

Another important result owed to him is the following conservativity of WKL_0.

Theorem 3 (Simpson, Tanaka and Y. 02)**.** *if* WKL_0 *proves* $\forall X \exists ! Y \varphi(X, Y)$ *with* φ *arithmetical, so does* RCA_0*.*

Memories: This statement was called Tanaka's conjecture. To show the conjecture, we tried various approarches and tried to find evidences. After we repeated trial and error, the conjecture was solved by Prof.Simpson's essential contribution in the end.

Tanaka's conjecture is not unrelated to the previous theorem. “the unique existence of Haar measure” looks of the form in Theorem 3. (Indeed it is not correct.)

With RCA_0, we prove that the real number system $\mathbb{R}$ satisfys the axioms of real closed ordered fields RCOF. However, this fact does not necessarily mean that the system $\mathbb{R}$ satisfies all the theorem of RCOF.

Simpson and Tanaka claimed with no details of the proof that within RCA_0, $\mathbb{R}$ does in fact satisfy the theorems of RCOF, with an appropriate notion of satisfaction. Then we basically adopt the definition of satisfaction relation $\mathrm{Sat}_{\mathbb{R}}$, which is given by quantifier elimination.

Theorem 4 (Tanaka and Y. 05). *Any provably total function in RCOF is also total in* $\mathbb{R}$ *via* $\mathrm{Sat}_{\mathbb{R}}$.

Later Tanaka and Sakamoto completed the proof.

Recent

Reverse mathematics on Brouwer's fixed-point theorem is also one of Tanaka's early result.

Theorem 5 (Shioji-Tanaka, 90)**.** *Brouwer fixed point theorem: "Let C be the convex hull of a non-empty finite subsets of $\mathbb{R}^n$. Then every continuous function $f : C \to C$ has a fixed point", is equivalent to* WKL_0 *over* RCA_0*.*

Recently, using this result, W.Peng and I showed:

1. A variant of Browder's fixed point theorem is equivalent to WKL_0 over RCA_0.
2. A variant of Kakutani's fixed point theorem is equivalent to ACA_0 over RCA_0.
3. ...

Supplement I

According to the number of papers and the period of research, his most important result is to check the proof theoretic strength for determinacy of many classes of infinite games.

Moreover, **many students** of him studied this topic.

1. Infinite games and transfinite recursion of multiple inductive definitions. CiE 2012, Springer LNCS,7318,(2012),374-383. With K. Yoshii.
2. Determinacy of Infinite Games and Inductive Definition in Second Order Arithmetic. 京都大学数理解析研究所講究録,1729,(2010),146-151. With K.Mashiko, K.Yoshii.
3. A note on multiple inductive definitions. Proceedings of the 10th Asian Logic Conference,(2010),345-352
4. ...

Although I am greatly influenced by him and I am a coauthor of 9 papers, I have not studied determinacy.

Recently I became interested in determinacy of Blackwell games and its extensions.

Then, I may have a chance to have my name entered in the list in the near future.

Supplement II

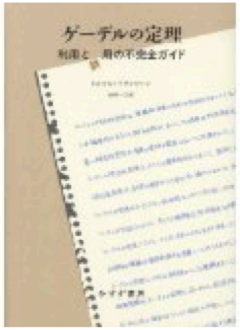
Professor Tanaka is also an excellent educator and populaizer!!

1. He has guided about 40 students (of course, including me!)
2. According to him, we form a big group.
3. He published 11 books (including 4 translations) and edited a serieses.


BLUE BACKS
ロジックの世界
ダン・クライアン／シャロン・シュアティル 文
ビル・メイブリン 絵　田中一之 訳


チューリングと超パズル
解ける問題と解けない問題
田中一之
このパズル、解けますか?


ゲーデルに挑む
証明不可能なことの証明
田中一之 著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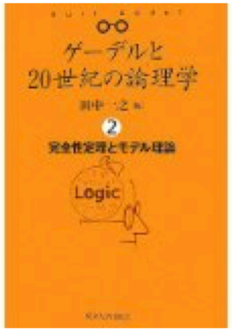
1
ゲーデルの20世紀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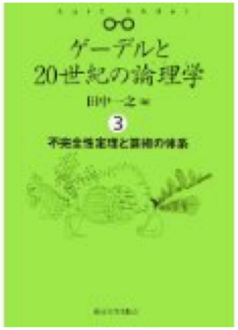
完全性定理とモデル理論
Logic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不完全性定理と算術の体系


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20世紀の論理学
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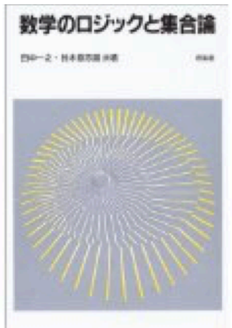
4
集合論とプラトニズム


数学の基礎についての哲学論考
確かさを求めて
THE SEARCH FOR CERTAINTY


The Mathematical
Intelligencer
数学の基礎をめぐる
論争
田中一之


数学のロジックと集合論


逆数学と2階算術
田中一之 著


数の体系と
超準モデル
田中一之 著


数学基礎論講義

Dear Professor Tanaka,
Congratulations on your 60th birthday!!