Computable Linear Orderings

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Spectra

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The spectrum of R is the collection of Turing degrees (perhaps others) of such $\hat{R}$s.

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Question: Does there exist a computable linear ordering with infinite successivities such that *Succ* is intrisically incomplete (i.e. the spectrum of *Succ* does not contain $\mathbf{0}'$)?

The answer is "yes" for wtt-degrees.

No for Turing degrees

Theorem: DLW

For any computable linear ordering with infinite successivities, $\mathcal{A}$, there is an isomorphic copy $\mathcal{B}$ such that $K \leq_T Succ(\mathcal{B})$ (i.e. $Succ(\mathcal{B})$ has Turing degree $\mathbf{0}'$).

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- At each stage s, between A_s and B_s, we only have a partial mapping.
- The final isomorphism can be read off from the true path of the construction tree.

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Thanks!